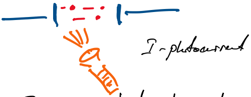
Inelastic scattering: role of energy and momentum

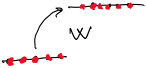
① Photoelectric effect



Two important observations:

1) ν_0 – threshold frequency
2) $E_{\bar{e}}$ increases linearly with ν

Einstein (1905)



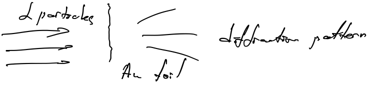
$$E_{\bar{e}} = \frac{1}{2} m v^2 = E_{ph} - W =$$

$$= h\nu - W$$

Millikan (1915) experimental demonstration

Yet it was not enough for Einstein and community.

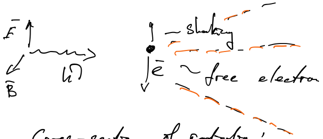
In 1911 Rutherford already made his electron scattering experiments.



It allowed to suggest electrostatic nature of the scattering and assume nucleus as a circular objects of diameter d.

=> Rutherford planetary model.

Can we see same picture for photon?



Cross-section of radiation:

$$\frac{d\sigma}{d\Omega} = \left(\frac{e^2}{mc^2}\right)\frac{1}{2}\left(1+\cos^2\theta\right)$$

area per solid angle

Important in Tomson scattering the ν_0 is the same as ν_s, like Rayleigh.

We need to come back to Einstein who had been „in a difficult struggle against quantum, pulsed by small happy interludes of some discoveries"

Einstein (1916) works on general relativity. He first admitted the „photons"

$$\swarrow \qquad \searrow$$

quanta of energy   quanta of momentum

$E^2 - p^2c^2 = m^2c^4$ - special relativity

$$\begin{cases} E = \dfrac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} \\ \bar{p} = \dfrac{m\bar{v}}{\sqrt{1-\frac{v^2}{c^2}}} \end{cases} \Rightarrow E^2 - p^2c^2 = m^2c^4$$

$$\Downarrow$$

So basically if you know the energy and the momentum you can know the particle mass.

This is relativistic version of what we new from Newton, i.e. non-relativistic

$$\left.\begin{aligned} E &= \frac{1}{2}mv^2 \\ \bar{p} &= m\bar{v} \end{aligned}\right\} \quad E = \frac{p^2}{2m}$$

Now, what about photons. Photons mass is 0, therefore

$$E_\gamma = p_\gamma c \qquad (m_\gamma = 0)$$

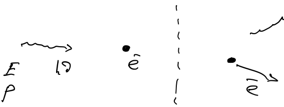
What is photon momentum?

$$p_\gamma = \frac{E_\gamma}{c} = \frac{h\nu}{c} = \frac{h}{\lambda_\gamma}$$

Now we are ready for Compton scattering: photon scattering on electrons that are „virtually" free. Compton found violation of classical Thomson scattering:

Compton found for higher energies the result is different.

Let's treat photon as particle, that has some energy and momentum



free photon cannot adsorb energy

All energy goes to kinetic, hence photon needs to lose energy $\Rightarrow \lambda_f > \lambda_i$

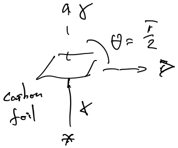
$$\boxed{\lambda_f = \lambda_i + \frac{h}{m_e c}(1-\cos\theta)}$$

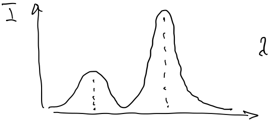
something with units of length and about $\bar{e}$.

At $\theta = \frac{\pi}{2}$ there is a $l_c(\bar{e})$ Compton shift.

X-ray $\lambda = 0.0709$ nm

$E_\gamma = 17.49$ keV





$\lambda_f - \lambda_i = 0.0022$ nm

but $l_c(\bar{e}) = \frac{h}{m_e c} =$

$\lambda_i = 0.0709$ nm $\lambda_f = 0.0731$ nm λ $= 2.426$ pm $=$

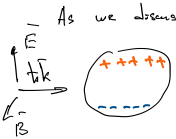
$= 0.0024$ nm

Compton (1923) was followed by Raman (1928)

Raman vs Mandelshtam and Landsberg

Mandelshtam - Landsberg scattering explanation

As we discussed:

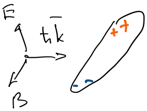


$\bar{\mu} = \hat{\chi}\bar{E}$

dipole moment

χ - polarizability tensor

Why we need tensor? Because matter can be anisotropic



$$\chi^k = \begin{pmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{pmatrix}$$ uniform and isotropic

$$\chi^a = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & k \end{pmatrix}$$ anisotropic

When previously we discussed resonance cases as μ large and $\frac{1}{E^2-(\hbar\omega)^2}$ (FGR)

Usually $\mu = \chi E + \underbrace{\chi^{(2)}EE^* + \ldots}_{\text{nonlinear optics}}$

(requires high fields and displacement)

But there can be always small displacement as long as we have field $E = E_0\cos(\omega_0 t)$

$\swarrow$

dipole approximation

$$\hat{\chi} = \hat{\chi}_{q=0} + \bar{q}\,\frac{\partial \hat{\chi}}{\partial q}\Big|_{q=0} + \dots$$

Taylor expansion

$$q = q_0 \cos \omega_v t$$ – harmonic oscillating displacement

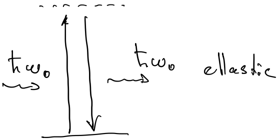
$$\bar{\mu} = \left(\hat{\chi}_{q=0} + \frac{\partial \hat{\chi}}{\partial q}\Big|_{q=0} q_0 \cos(\omega_v t)\right) \bar{E}_0 \cos(\omega_0 t)$$

$$2 \cos a \cos b = \cos(a+b) + \cos(a-b)$$

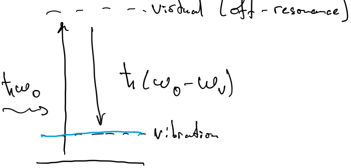
$$\bar{\mu} = \hat{\chi}_{q=0} \bar{E}_0 \cos(\omega_0 t) + \frac{1}{2} q_0 \frac{\partial \hat{\chi}}{\partial q}\Big|_{q=0} E_0 \left(\cos(\omega_0 - \omega_v)t + \cos(\omega_0 + \omega_v)t\right)$$

1 2 3

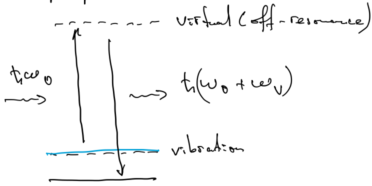
1) $\chi_{q=0} \bar{E}_0 \cos \omega_0 t$ – Rayleigh-Tyndall



2) $\frac{1}{2} q_0 \frac{\partial \hat{\chi}}{\partial q}\Big|_{q=0} \bar{E}_0 \cos(\omega_0 - \omega_v)t$ – Stokes (Raman)



3) $\frac{1}{2} q_0 \frac{\partial \chi}{\partial q}\Big|_{q=0} \bar{E}_0 \cos(\omega_0+\omega_v)t$ - anti - Stokes



Mandelshtam and Landsberg gave most broad explanation of the effect on both energy sides, calling it Combinational scattering.

Long story short:

$$I \approx |g_s \cdot \alpha(j) \cdot g_i|^2$$

polarization

$$\alpha_i \sim \sum_{k,l} \frac{\partial \chi}{\partial q}$$ ~ hence it requires xy, z^2, x^2 and other group symm.

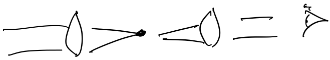
power of 2

Example: methanol and ethanol.

molecular fingerprint

The idea of concentrating light:

1) Seeing smaller things:



2) Concentrating field in smaller volume -> field enhancement.

What is used in radio-waves? Antenna

Hertz experiment

AN-TENN-A

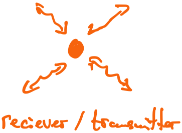
↙ "up"

↘ Indo-European: "to stretch"

tension (English)
tan (Sanskrit)
tendere (Latin)
teinei (Greek)
dehnen (German)
tyanut (Russian)
taanna (Hindi)

Antenna – a device that catches e/m wave.

Optical antenna – a device to convert optical radiation to localized energy, and vice versa.



1) Edward Synge and Albert Einstein
2) Light confined at nanoscale
plasmonics and beyond
3) Revisit of Hertz experiment and
electron at nanoscale

Compton with visible light.